AP Saturday Boot Camp, April 26, 2025

26. $\lim_{x\to\infty} \frac{\ln(e^{3x}+x)}{x} = \lim_{x\to\infty} \frac{\frac{1}{e^{3x}+x}\cdot 3e^{3x}+1}{1} = \lim_{x\to\infty} \frac{3e^{3x}+1}{e^{3x}+\cancel{x}} = \lim_{x\to\infty} \frac{9e^{3x}}{3e^{3x}+\cancel{x}} = \frac{9e^{3x}}{3e^{3x}}$

(A) 0

(B) 1

(C) 3

(D) ∞

$\frac{\infty}{\infty}$

$\frac{\infty}{\infty} ? \frac{0}{0}$

The weight of a population of yeast is given by a differentiable function y, where $y(t)$ is measured in grams and t is measured in days. The weight of the yeast population increases according to the equation $\frac{dy}{dt} = ky$, where k is a constant. At time $t = 0$, the weight of the yeast population is 120 grams and is increasing at the rate of 24 grams per day.

14. Which of the following is an expression for $y(t)$?

(A) $120e^{24t}$

(B) $120e^{t/5}$ TEST

(C) $e^{t/5} + \cancel{119}$

(D) $\cancel{24t} + 120$

$X^2 \cdot X^3 = X^{2+3} = X^5$

$\frac{dy}{dT} = Ky$

$\int \frac{1}{y} dy = \int K\,dT$

$T = 1$

$Y = 144$

$24 = K\cancel{y}$

$e^{KT} \cdot e^{C} = e^{KT+C}$

$\ln|y| = KT + C$

$\frac{120}{24} = 5$

$(e^{K})^{T} \cdot 120 = y$

$e^{KT+C} = y$

$(\frac{6}{5})^{T} \cdot 120 = y$

$e^{KT} \cdot 120 = y$

$e^{KT} \cdot e^{C} = y$

$e^{K\cdot 0} \cdot e^{C} = 120$

$120 e^{0/5} = 120$

$\frac{120 e^{K\cdot 1}}{120} = \frac{144}{120}$

$e^{0} \cdot e^{C} = 120$

$120 e^{1/5} = 144$

$e^{K} = \frac{12}{10} = \frac{6}{5}$

$1 \cdot e^{C} = 120$

$e^{C} = 120$

$Y = 120$

$e = 2.71$

$e^{24\cdot 1}$

$24 = Ky$

$e^{K} \cdot e^{\frac{T}{24}}$

$e^{K} = \frac{6}{5}$

$e^{\frac{KT}{24}} = y$

$24 \ln y = KT$

$\ln y = \frac{KT}{24}$

$\frac{24}{120} = K$

$\frac{1}{5} = K$

30. For a certain continuous function *f*, the right Riemann sum approximation of $\int_0^2 f(x)\,dx$ with *n* subintervals of equal length is $\frac{2(n+1)(3n+2)}{n^2}$ for all *n*. What is the value of $\int_0^2 f(x)\,dx$?

$n = \infty$

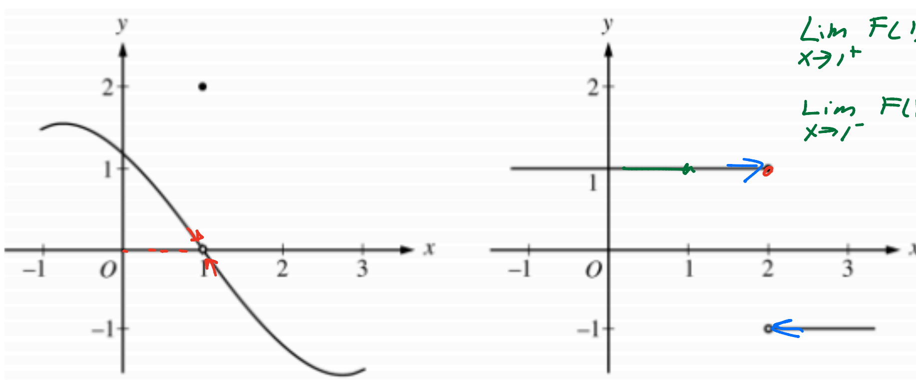
(A) 2

(B) 6

(C) 12

(D) 20

$\lim_{n \to \infty} \frac{2(n+1)(3n+2)}{n^2} = \frac{6n^2}{n^2} = 6$



Graph of *f*   Graph of *g*

$\lim_{x \to 1^+} F(1) \cdot g(2.1) = 0 \cdot 1 = 0$

$\lim_{x \to 1^-} F(1) \cdot g(1.99) = 0 \cdot -1 = 0$

The graphs of the functions *f* and *g* are shown in the figures above.

15. Which of the following statements is false?

(A) $\lim_{x \to 1} f(x) = 0$ True

(B) $\lim_{x \to 2} g(x)$ does not exist. different $\lim_{x \to 2^+} g(x) \neq \lim_{x \to 2^-} g(x)$

(C) $\lim_{x \to 1} (f(x)g(x+1))$ does not exist.

(D) $\lim_{x \to 1} (f(x+1)g(x))$ exists. $= \cancel{\lim} F(2) \cdot g(1) = 1 \cdot 1 = $ EXISTS

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6. If *f* is the function given by f(x) = $3x^2 - x^3$, then the average rate of change of *f* on the closed interval [1, 5] is

(A) -21
(B) -13
(C) -12
(D) -9

$F(1) = 3 - 1 = 2$

$F(5) = 3 \cdot 25 - 125 = -50$

$$\frac{-50-2}{5-1} = \frac{-52}{4} = -13$$

20. Let *f* be the function given by $f(x) = \frac{x-2}{2|x-2|}$. Which of the following is true?

(A) $\cancel{\lim_{x\to 2} f(x)} = \frac{1}{2}$

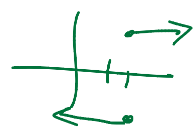
Hole ra

(B) *f* has a removable discontinuity ~~at x~~ = 2. Just a dot

(C) *f* has a jump discontinuity at *x* = 2 .

(D) *f* has a discontinuity due to a vertical ~~asymptote~~ at *x* = 2.

$$\frac{1.99-2}{2|1.99-2|} = -\frac{1}{2}$$



$$\frac{2.01-2}{2(2.01-2)} = \frac{1}{2}$$

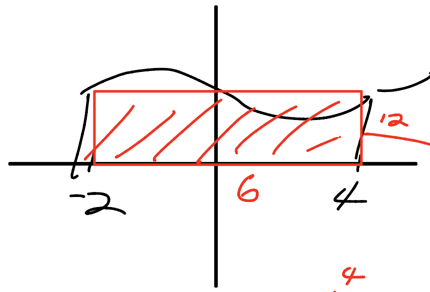
24. If the average value of a continuous function f on the interval $[-2, 4]$ is 12, what is $\int_{-2}^{4} \frac{f(x)}{8}\,dx$?

(A) $\frac{3}{2}$

(B) 3

(C) 9

(D) 72



$\int_{-2}^{4} F(x)\,dx = \text{Average value} \cdot 6$

average value = height of Rectangle

$12 \cdot 6 = 72 = \int_{-2}^{4} F(x)\,dx$

$\int_{-2}^{4} \frac{F(x)}{8}\,dx = \frac{1}{8}\int_{-2}^{4} F(x)\,dx = \frac{1}{8} \cdot 72$

$= 9$

17. A particle moves along the x-axis so that at time $t > 0$ its position is given by $x(t) = 12e^{-t}\sin t$. What is the first time t at which the velocity of the particle is zero?

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$

(D) π

$\cos\frac{\pi}{4} - \sin\frac{\pi}{4} = 0$

$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$

$V(T) = X'(T) = 12 \cdot -1 \cdot e^{-T} \cdot \sin T + 12e^{-T} \cdot \cos T$

$= 12e^{-1}(-\sin T + \cos T)$

First Time equal

$\frac{5\pi}{4}$ = second Time equal

$x = T \qquad dx = dT$

18. Let F be the function given by $F(x) = \int_3^x (\tan(5t)\sec(5t) - 1)\,dt$. Which of the following is an expression for $F'(x)$?

$(\tan 5T \sec 5T - 1)$

(A) $\frac{1}{5}\sec(5x) - 1$

(B) $\frac{1}{5}\sec(5x) - x$

(C) $\tan(5x)\sec(5x)$

(D) $\tan(5x)\sec(5x) - 1$

8. If f is the function given by $f(x) = e^{x/3}$, which of the following is an equation of the line tangent to the graph of f at the point $(3 \ln 4, 4)$?

(A) $y - 4 = \frac{4}{3}(x - 3\ln 4)$

(B) $y - 4 = 4(x - 3\ln 4)$

(C) $y - 4 = 12(x - 3\ln 4)$

(D) $y - 3\ln 4 = 4(x - 4)$

$F'(x) = \text{Slope}$

$F'(x) = e^{\frac{x}{3}} \cdot \frac{1}{3}$

$F'(3\ln 4) = e^{\frac{3\ln 4}{3}} \cdot \frac{1}{3} = e^{\ln 4} \cdot \frac{1}{3} = 4 \cdot \frac{1}{3} = \frac{4}{3}$

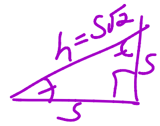
$\text{Slope} = \frac{4}{3}$

$y - 4 = \frac{4}{3}(x - 3\ln 4)$

28. At what rate is the length of the hypotenuse of the triangle increasing, in centimeters per second, at that instant?

An isosceles right triangle with legs of length s has area $A = \frac{1}{2}s^2$. At the instant when $s = \sqrt{32}$ centimeters, the area of the triangle is increasing at a rate of 12 square centimeters per second.

$12 = \frac{dA}{dT}$



$A = \frac{1}{2} \cdot s^2$

(A) $\frac{3}{4}$

(B) 3

(C) $\sqrt{32}$

(D) 48

$\sqrt{2} \cdot \sqrt{32} = \frac{h}{\sqrt{2}} \cdot \sqrt{2}$

$\sqrt{64} = h$

$8 = h$

$h = s\sqrt{2}$

$\frac{h}{\sqrt{2}} = s$

$A = \frac{1}{2} \cdot s^2$

$A = \frac{1}{2} \cdot \left(\frac{h}{\sqrt{2}}\right)^2$

$A = \frac{1}{4} h^2$

$\frac{dA}{dT} = \frac{1}{4} \cdot 2h \cdot \frac{dh}{dT}$

$12 = \frac{1}{4} \cdot 2 \cdot 8 \frac{dh}{dT}$

$12 = 4 \frac{dh}{dT}$

$\frac{dh}{dT} = 3$

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10. $\int_0^2 (x^3+1)^{1/2} x^2 \, dx =$

(A) $\frac{52}{9}$

(B) 6

(C) $\frac{26}{3}$

(D) $\frac{52}{3}$

$u = x^3 + 1$

$du = 3x^2 dx$

$\frac{du}{3x^2} = dx$

$\int u^{\frac{1}{2}} \cdot \cancel{x^2} \cdot \frac{du}{3\cancel{x^2}}$

$\frac{1}{3} \int u^{\frac{1}{2}} du$

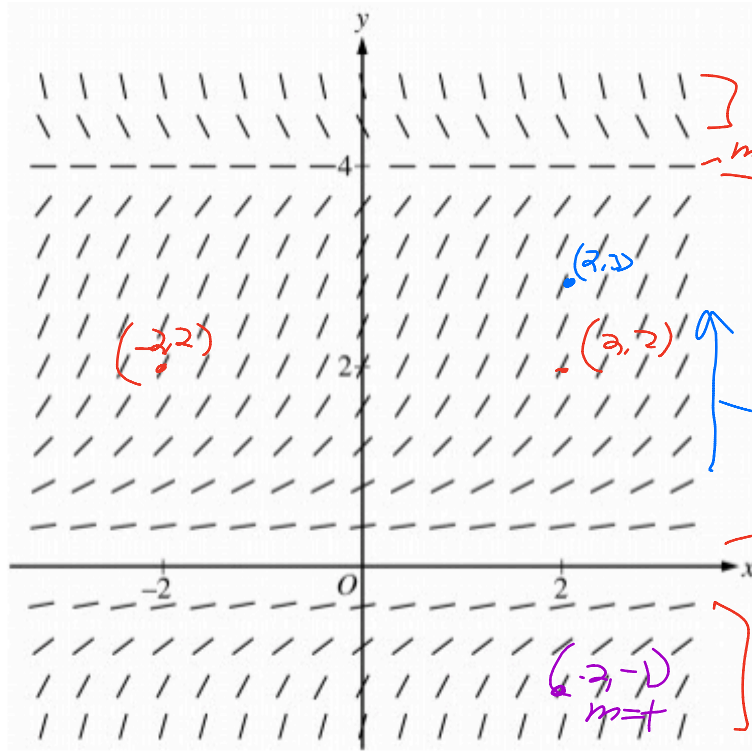
$\frac{1}{3} \cdot \frac{2}{3} \cdot u^{\frac{1}{2}+1=\frac{3}{2}} + C$

$\frac{2}{9}(x^3+1)^{3/2} + C \Big[_0^2$

$(8+1)^{3/2} = (\sqrt{9})^3 = 27$

$\frac{2}{9}(2^3+1)^{3/2} - \frac{2}{9}(0^3+1)^{3/2}$

$\frac{2}{9} \cdot 27 - \frac{2}{9} \cdot 1 = \frac{2}{9}(27-1) = \frac{2}{9} \cdot 26 = \frac{52}{9}$



13. Shown is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{\cancel{x}(4-y)}{4}$ $\quad \frac{-2(4-2)}{4} = \frac{-4}{4} = -1$

(B) $\frac{dy}{dx} = \frac{y(4-y)}{4}$ $\quad \frac{-2(4--2)}{4} = \frac{-16}{4} = -4$ $\quad \frac{2(4-2)}{4} = \frac{4}{4} = 1$ $\quad \frac{3(4-3)}{4} = \frac{3}{4}$ goes down

(C) $\frac{dy}{dx} = \cancel{\frac{xy(4-y)}{4}}$ $\quad \frac{-2(2)(4-2)}{4} = \frac{-8}{4} = -2$

(D) $\frac{dy}{dx} = \frac{y^2(4-y)}{4}$ $\quad \frac{(-2)^2(4-2)}{4} = \frac{24}{4} = 6$ $\quad \frac{2^2(4-2)}{4} = 2$ $\quad \frac{9(4-3)}{4} = \frac{9}{4} = 2\frac{1}{4}$ goes up

22. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{2y}{2x+1}$ with the initial condition $y(0) = e$ for $x > -\frac{1}{2}$?

(A) $y = e^{2x^2+2x+1}$

(B) $y = 2ex + e$

(C) $y = \sqrt{x^2 + x + e^2}$

(D) $y = \sqrt{\frac{1}{2}\ln(2x+1) + e^2}$

$(0, e)$

$\int \frac{1}{2y}\,dy = \int \frac{1}{2x+1}\,dx$

$\frac{1}{2}\ln y = \frac{1}{2}\ln|2x+1| + C$

$\frac{1}{2}\ln e = \frac{1}{2}\ln|2\cdot 0+1| + C$

$\frac{1}{2}\cdot 1 = \frac{1}{2}\cdot 0 + C$

$\frac{1}{2} = C$

$2\cdot\left(\frac{1}{2}\ln y\right) = \left(\frac{1}{2}\ln|2x+1| + \frac{1}{2}\right)\cdot 2$

$\ln y = \ln|2x+1| + 1$

$e^{\ln|2x+1| + 1} = y$

$e^{\ln 2x+1}\cdot e^{1} = y$

$(2x+1)\,e^{1} = y$

$\int \frac{1}{2x+1}\,dx = \int \frac{1}{u}\cdot\frac{du}{2} = \frac{1}{2}\int \frac{1}{u}\,du$

$u = 2x+1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\frac{1}{2}\ln|u| + C$

$\frac{1}{2}\ln|2x+1| + C$

29. The graph of which of the following functions has exactly one horizontal asymptote and no vertical asymptotes?

(A) $y = \frac{1}{x^2+1}$

(B) $y = \frac{1}{x^3+1}$

(C) $y = \frac{1}{e^x-1}$

(D) $\cancel{y = \frac{1}{e^x+1}}$

$x = -1$ $\quad (-1)^3 + 1 = 0$

$x = 0$ $\quad e^0 - 1 = 1 - 1 = 0$

$y = \frac{1}{e^x+1} = (e^x+1)^{-1}$

$\frac{dy}{dx} = -1(e^x+1)\cdot e^x = \frac{-e^x}{(e^x+1)^2} \neq 0$

$\frac{dy}{dx} = 0$ $\quad \frac{k}{0}$ $\quad \frac{dy}{dx} = \emptyset$

$y = \frac{1}{x^2+1} = (x^2+1)^{-1}$

$\frac{dy}{dx} = -1(x^2+1)^{-2}\cdot 2x = \frac{-2x}{(x^2+1)^2} \pm \emptyset$

slope

$e^x \neq$ For all x

The region enclosed by the graphs of $y = x^2$ and $y = 2x$ is the base of a solid. For the solid, each cross section perpendicular to the y-axis is a rectangle whose height is 3 times its base in the xy-plane.

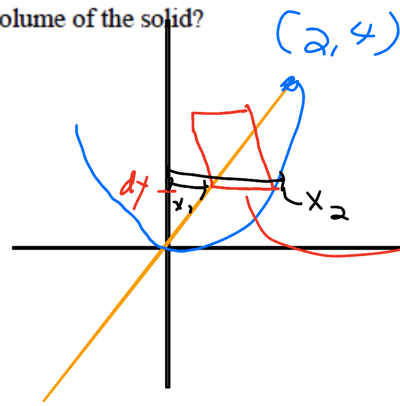
23. Which of the following expressions gives the volume of the solid?

(A) $3\int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$

(B) $3\int_0^4 \left(\sqrt{y} + \frac{y}{2}\right)^2 dy$

(C) ~~$3\int_0^2 (2x - x^2)^2 dx$~~

(D) ~~$3\int_0^2 (2x + x^2)^2 dx$~~



$\int_0^4$

base $(x_2 - x_1)$

$\left(\sqrt{y} - \frac{y}{2}\right)$ base

$y = x^2$ $\quad$ $y = 2x$

$\sqrt{y} = x_2$ $\quad$ $\frac{y}{2} = x_1$

$\int_0^4 \left(\sqrt{y} - \frac{y}{2}\right) \cdot 3\left(\sqrt{y} - \frac{y}{2}\right) dy$